3.1 Acute Angle Triangles

Acute Angle Triangles [3.1]

θ : theta

Trigonometry

Trigonometry means "three angle measurement". It is a branch of mathematics which deals with the measurement of angles and sides of triangles.

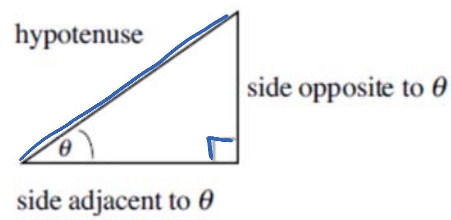
Primary Trigonometric Ratios

Complete the following

sine $\Rightarrow \sin\theta = \dfrac{op}{hy}$

cosine $\Rightarrow \cos\theta = \dfrac{adj}{hyp}$

tangent $\Rightarrow \tan\theta = \dfrac{op}{adj}$



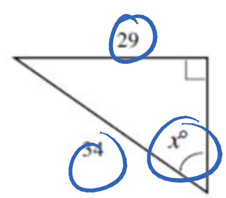
These ratios are called the ***Primary Trigonometric Ratios*** and can be remembered by the acronym **SOHCAHTOA**.

$\boxed{\text{Deg}}$

Use the diagram to determine

a) $\sin x° = \dfrac{29}{34} = 0.8529$

b) $x°$ $\sin^{-1}(0.8529) = 58.5°$

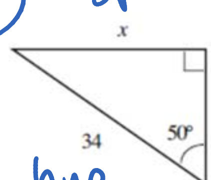


In each diagram find the length of the indicated side to the nearest tenth.

height of flagpole

a)

op



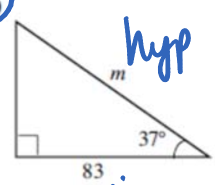
hyp

$34 \cdot \sin 50° = \dfrac{x}{\cancel{34}} \cdot \cancel{34}$

$34 \cdot \sin 50° = x$

b)

hyp



adj

$\cos 37° = \dfrac{83}{m}$

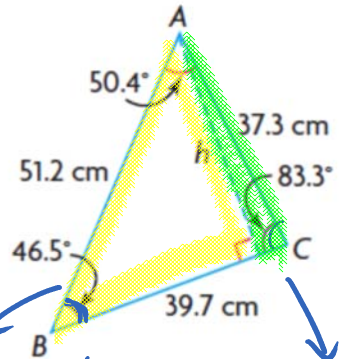
$\dfrac{m \cancel{\cos 37°}}{\cancel{\cos 37°}} = \dfrac{83}{\cos 37°}$

$m = \dfrac{83}{\cos 37°}$

$= 103.9$

$26.0 =$

Ex: What are two equivalent expressions that represent the height of Δ ABC? using the general Δ



$\sin 46.5^\circ = \frac{h}{51.2}$

$51.2(\sin 46.5^\circ) = h$

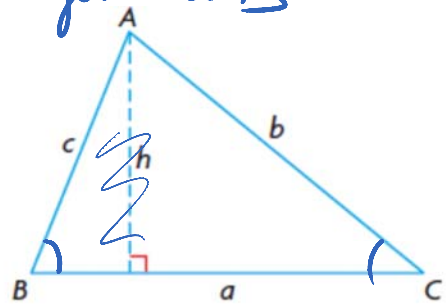
$h = 37.1\text{ cm}$

$\sin 83.3 = \frac{h}{37.3}$

$37.3(\sin 83.3^\circ) = h$

$h = 37.0\text{ cm}$

rounding errors



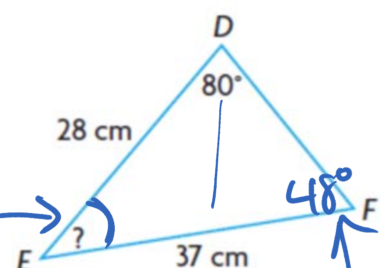
$\sin B = \frac{h}{c}$ | $\sin C = \frac{h}{b}$

$c \sin B = h = b \sin C = h$

$c \sin B = b \sin C$

$\boxed{\frac{\sin B}{b} = \frac{\sin C}{c}}$ Sine Law

Ex: Explain how you could determine the measure of $\angle$ E in this acute triangle.



① Find F

$\frac{\sin 80^\circ}{37\text{cm}} = \frac{\sin F}{28\text{cm}}$

$\frac{28 \sin 80^\circ}{37} = \sin F = 0.7455$

$F = \sin^{-1} 0.7455$

$= \underline{\underline{48^\circ}}$

② $180 - 80^\circ - 48^\circ$

$E = 52^\circ$

***In groups of 2/3 complete the "Getting Started Activity on page 114 on your Text.**

Hand in your estimates as you leave ☺

Practice

Practice

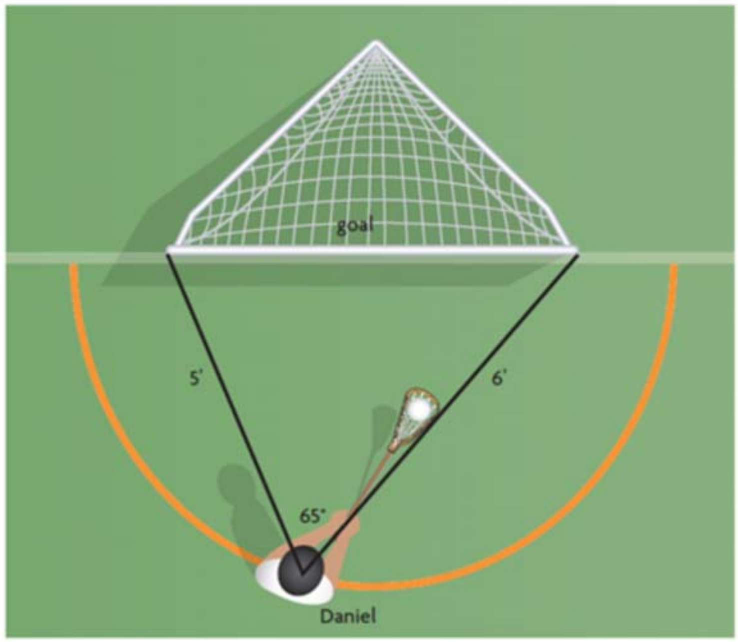
pg 117 #1, 2, 4

pg 114 do this activity on the next 2 pages of your notes "Getting Started"

3.1 Lacrosse Net

Getting Started Activity: The Lacrosse Net [3.1]

* Read through the entire activity on p. 114 <u>before</u> getting started in your small group of 2 or 3 *



Daniel is about to take a shot at a field lacrosse net. He estimates his current position, as above.

a) Does Daniel's position form a right triangle with the goalpost?

b) A primary trigonometric ratio cannot be used to determine the width of the net directly. Explain why.

c) Copy the triangle that includes Daniel's position in the diagram above. Add a line segment so that you can determine a height of the triangle using trigonometry.

d) Determine the height of the triangle using a primary trig ratio.

e) Create a plan that will allow you to determine the width of the lacrosse net using the two right triangles you created.

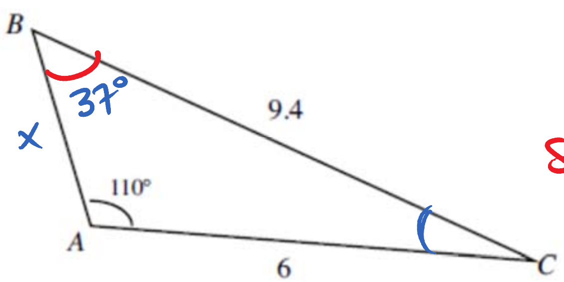
f) Carry out your plan to determine the width of the net.

3.2 Sine Law

Sine Law [3.2]

In every triangle ABC, $\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \boxed{\frac{\sin B}{b} = \frac{\sin C}{c}}$

Use the sine law in the triangle shown below to determine the measure of ∠ACB to the nearest degree.



$\frac{\sin C}{x} = \frac{\sin 110°}{9.4}$ not enough info

So solve for B first

$\frac{\sin B}{6} = \frac{\sin 110°}{9.4}$

$\sin B = \frac{6 \cdot \sin 110}{9.4} = 0.5998$

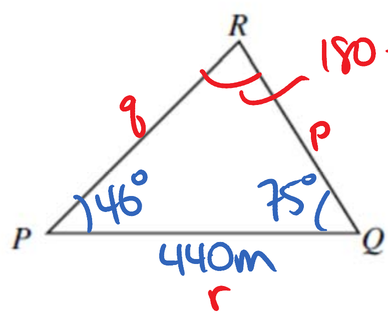
$B = \sin^{-1}(0.5998) = 37°$

$180° - 37° - 110° = C$

$C = 33°$

A surveyor measures a base line PQ 440m long. He takes measurements of a landmark R from P and Q, and finds that ∠ QPR = 46° and ∠ PQR = 75°.

a) Calculate the perimeter of ΔPQR to the nearest metre.



$180 - 46 - 75 = 59°$

$\frac{p}{\sin P} = \frac{r}{\sin R}$

$\frac{p}{\sin 46°} = \frac{440}{\sin 59°}$

$p = \frac{440 \cdot \sin 46°}{\sin 59°}$

$= 369.25$

$\frac{q}{\sin 75°} = \frac{440}{\sin 59°}$

$q = 495.85m$

Perimeter $= 440m + 369.25m + 495.85m$

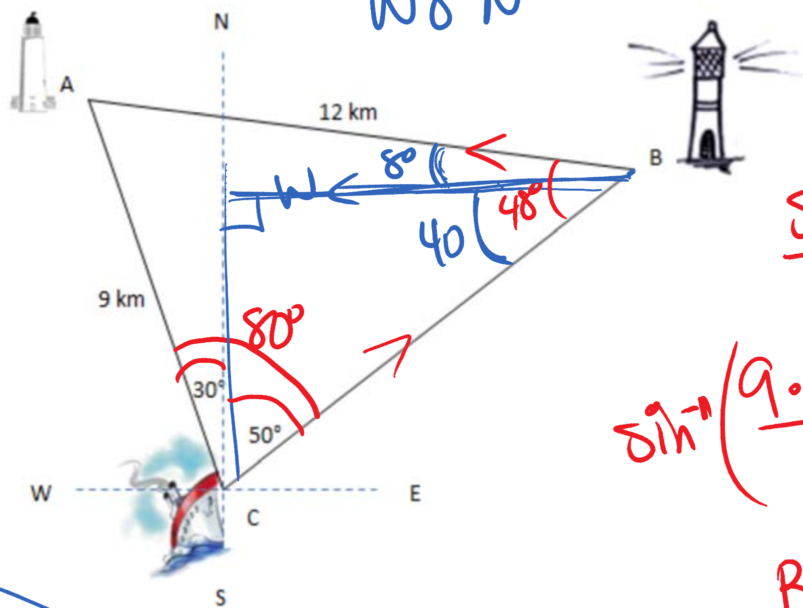
$= 1305m$

distance around Δ

distance around Δ $= 1305\text{m}$

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at N30°W and the lighthouse to his right is located at N50°E. determine the compass direction he must follow when he leaves lighthouse B for lighthouse A.

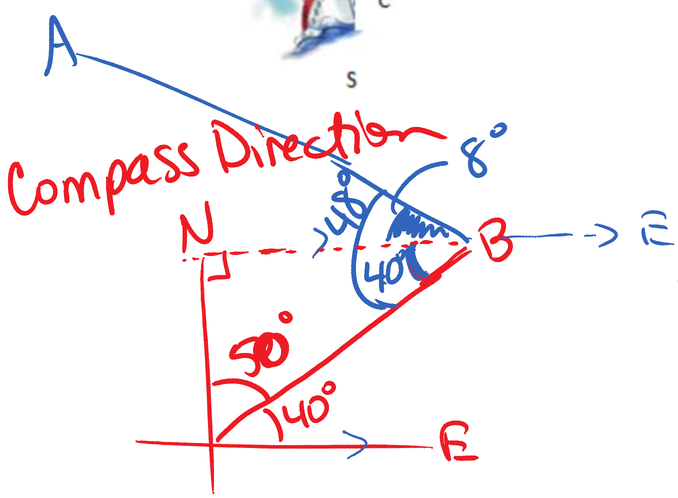
W8°N



$$\frac{\sin 80}{12} = \frac{\sin B}{9}$$

$$\sin^{-1}\left(\frac{9 \cdot \sin 80}{12}\right) = B$$

$$B = 47.6° = 48°$$

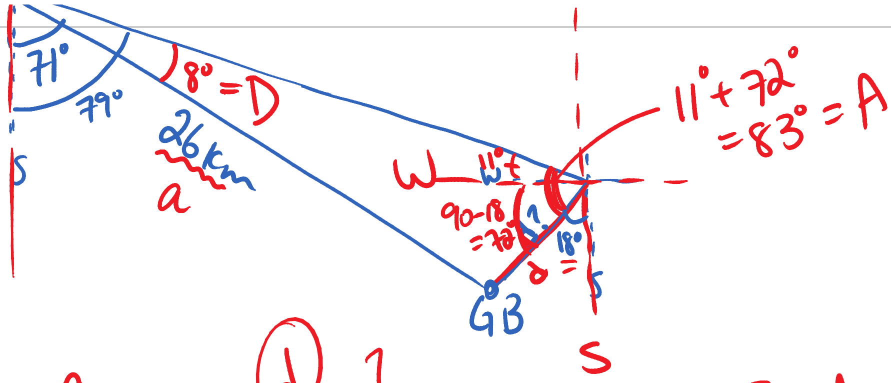


$\boxed{W8°N}$

$\boxed{N82°W}$

Practice pg 125 # 3, 8, 9, 11

#9 Gimli



$$\frac{a}{\sin A} = \frac{d}{\sin D}$$

$$\frac{26}{\sin 83^\circ} = \frac{d}{\sin 8^\circ}$$

$$\left(\frac{26}{\sin 83}\right)\sin 8 \quad \frac{26 \cdot \sin 8^\circ}{\sin 83^\circ} = d = 3.6 \text{ km}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{c \sin A}{a} = \sin C$$

$$\sin^{-1}(\ ) = C$$

Mid Ch Review Homework

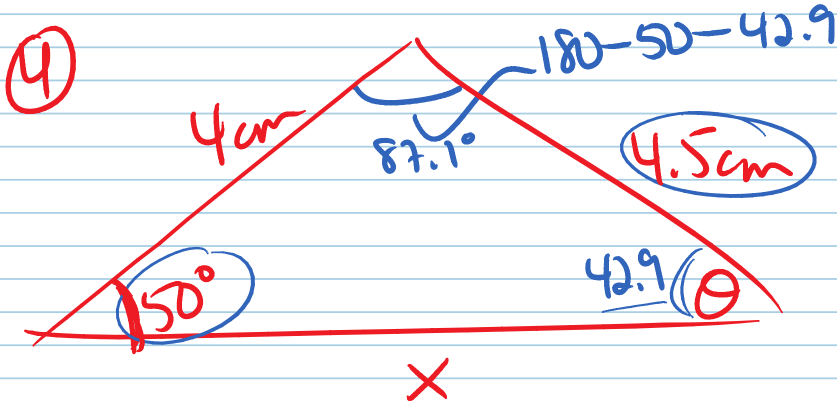
1. $z = 38°$ ✓

2. $y = 23.0 \text{ cm}$ ✓

3. 316m ✓, 300m ✓

More practice: pg 129 # 2–9

(4)



$$\frac{\sin\theta}{4} = \frac{\sin 50°}{4.5}$$

$$\sin\theta = \frac{4\sin 50°}{4.5}$$

$$\theta = \sin^{-1}\left(\quad\right)$$

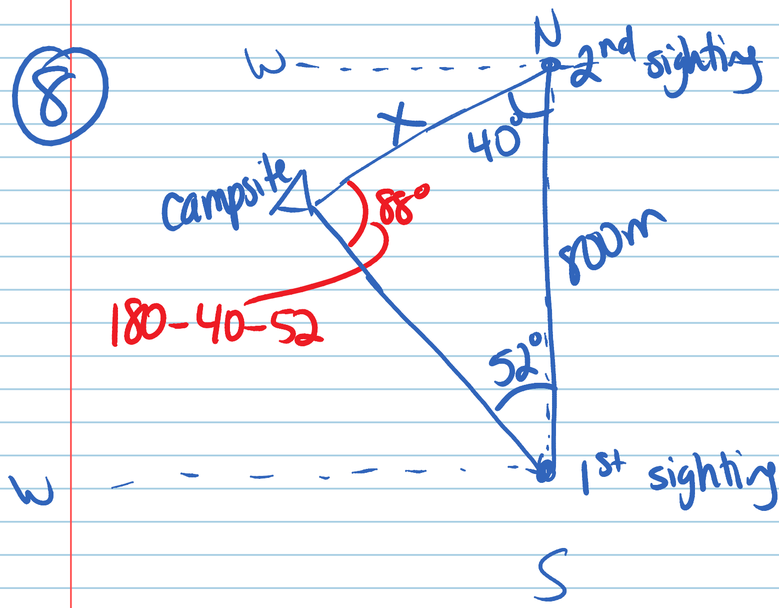
$$\theta = 42.9°$$

$$\frac{x}{\sin 87.1°} = \frac{4.5}{\sin 50°}$$

$$x = \frac{4.5\sin 87.1}{\sin 50}$$

$$= 5.9 \text{ cm}$$

8

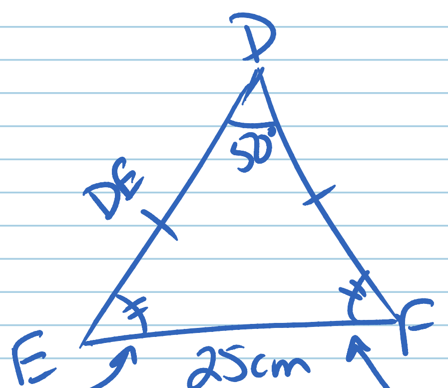


$$\frac{x}{\sin 52^\circ} = \frac{800}{\sin 88^\circ}$$

$$x = \frac{800 \sin 52^\circ}{\sin 88^\circ}$$

$$= 631\,m$$

9a)



$180 - 50 = 130$

$\frac{130}{2} = 65^\circ$

Perimeter $= 25 + DE + DF$

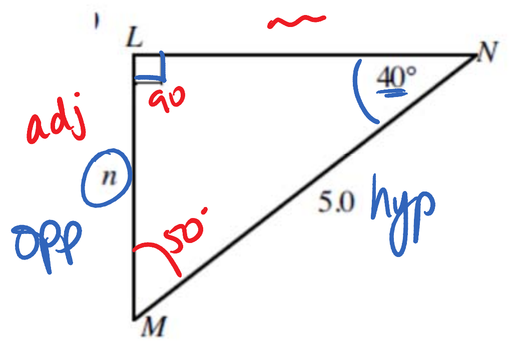
$$\frac{25cm}{\sin 50^\circ} = \frac{DE}{\sin 65^\circ}$$

$$DE = \frac{25 \sin 65}{\sin 50}$$

$$= \boxed{}$$

Cosine Law [3.3]

Quick Review: Determine the length of the indicated side to the nearest tenth.



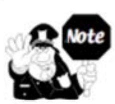
$\sin\theta = \frac{opp}{hyp}$

$\sin 40° = \frac{n}{5.0}$

$n = 5\sin 40°$

$= 3.2$

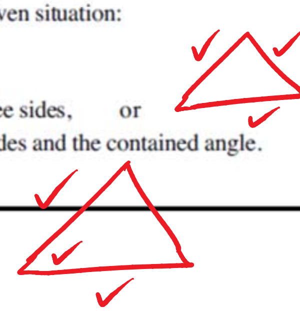
$\cos 50° = \frac{n}{5.0} \rightarrow n = 3.2$



To solve for an angle or a side in a triangle using trigonometry, three pieces of information are required, one of which must be the length of a side.

Use the following guide to determine which law to use in any given situation:

- In a right-angled triangle use SOHCAHTOA
- Use Law of Cosines if you are given either; i) all three sides, or ii) two sides and the contained angle.
- In all other cases use the Law of Sines.



$a^2 + b^2 = c^2$

$c^2 = a^2 + b^2 - 2ab\cos C$

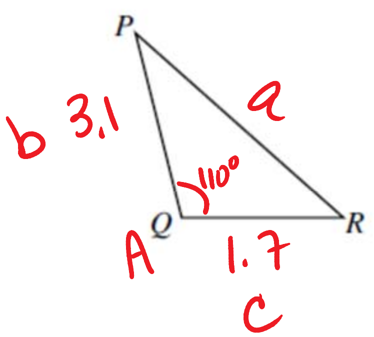
$b^2 = a^2 + c^2 - 2ac\cos B$

The equation $\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

can be rearranged to the form $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$. ✓

This form of the cosine law can be used to determine any angle in a triangle when we are given the length of all three sides, (SSS) + (SAS)

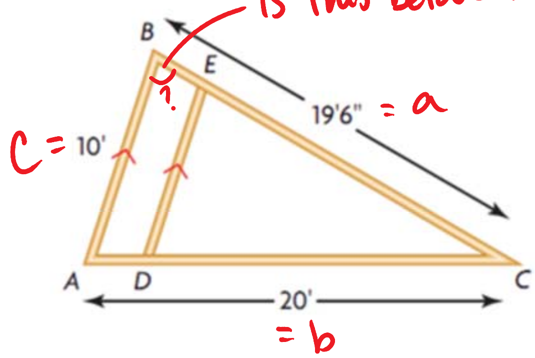
Find the length, to the nearest tenth of a cm, of the third side of Δ PQR if QP = 3.1cm, QR = 1.7 cm and $\angle$ PQR = 110°.



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = (3.1)^2 + (1.7)^2 - 2(3.1)(1.7)\cos 110°$$
$$a^2 = 16.105$$
$$a = 4.0$$

The diagram shows the plan for a roof, with support beam DE parallel to AB. The local building code requires the angle formed at the peak of a roof to fall within a range of 70° to 80° so that snow and ice will not build up. Will this pass the local building code?

Is this between 70° and 80°



given all sides, solve for angle

$$\frac{6''}{12''} = 0.5$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$\frac{b^2 - a^2 - c^2}{-2ac} = \frac{\cancel{-2ac}\cos B}{\cancel{-2ac}}$$
$$\frac{b^2 - a^2 - c^2}{-2ac} = \cos B$$
$$\frac{a^2 + c^2 - b^2}{2ac} = \cos B$$
$$\frac{19.5^2 + 10^2 - 20^2}{2(19.5)(10)} = \cos B$$

$$\frac{19.5^2 + 10^2 - 20^2}{2(19.5)(10)} = \cos B$$

$$\frac{80.25}{390} = $$

$$0.2058 = \cos B$$

$$B = \cos^{-1}(0.2058)$$

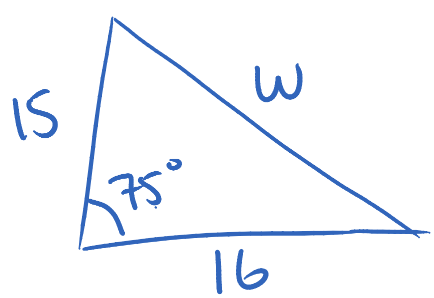
$$B = 78°$$

yes will pass building code.

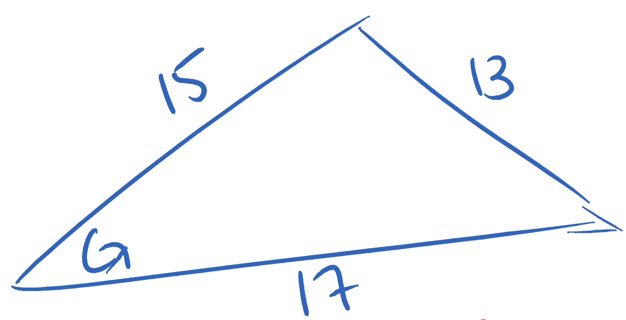
Practice pg 137 #4-6, 8, 11

$a^2 = b^2 + c^2 - 2bc \cos A$.

6 a) $w^2 = 15^2 + 16^2 - 2(15)(16)\cos 75°$



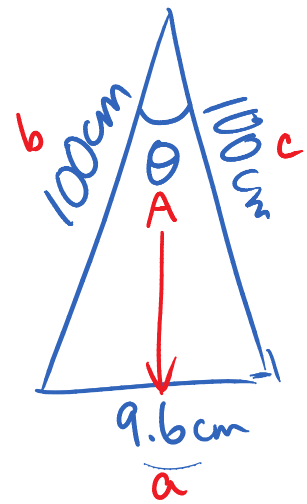
d) $13^2 = 17^2 + 15^2 - 2(17)(15)\cos G$



$a^2 = b^2 + c^2 - 2bc \cos A$

17

8.



$a^2 = b^2 + c^2 - 2bc\cos A$

$9.6^2 = 100^2 + 100^2 - 2(100)(100)\cos\theta$ ✓

$92.16 = \underbrace{10000 + 10000}_{20000} - 20000\cos\theta$

$92.16 - 20000 = -20000\cos\theta$

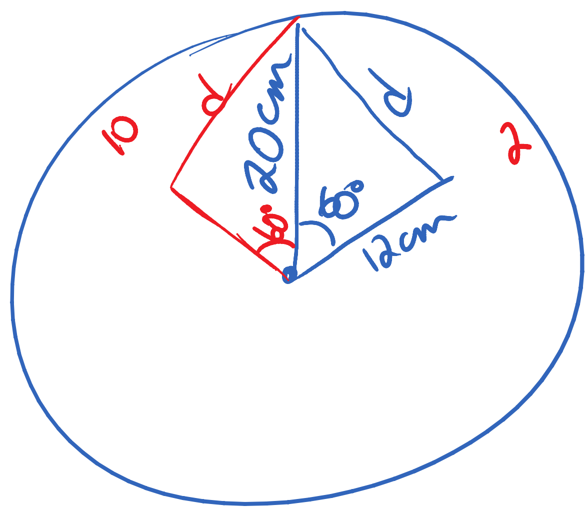
$\frac{-19907.84}{-20000} = \frac{\cancel{-20000}\cos\theta}{\cancel{-20000}}$

$0.9954 = \cos\theta$

$\cos^{-1}0.9954 = \theta$

$5.5° = \theta$

(11)



$\frac{360°}{12\text{ hrs}} = 30°$ per hour

$2\text{ hrs} \to 60°$

$d^2 = 12^2 + 20^2 - 2(12)(20)\cos 60°$

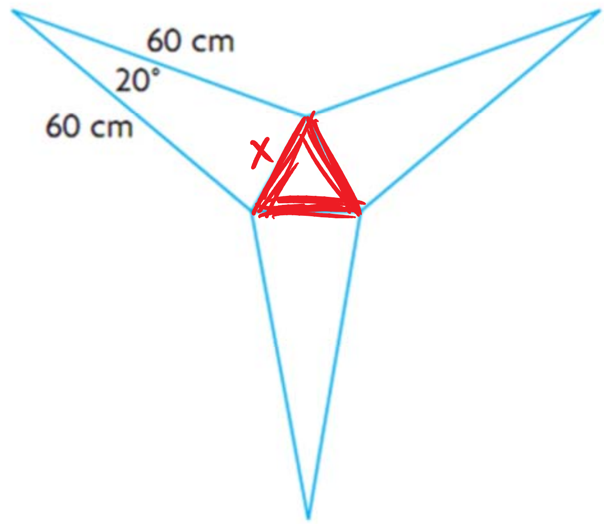
$d^2 = 304$

$d = d = 17.4\text{cm}$

3.4 (1) Problem Solving

Problems Involving Acute Triangles – Part 1 [3.4]

Warming up: A three-pointed star is made up of an equilateral triangle and three congruent isosceles triangles. Determine the length of each side of the equilateral triangle in this three-pointed star. Round the length to the nearest centimetre.

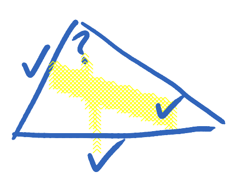
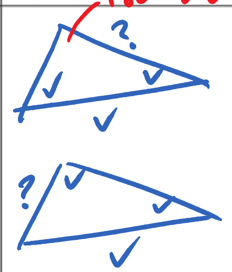
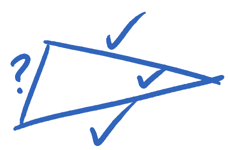
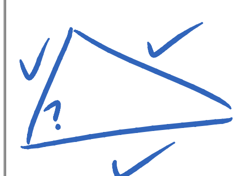


$$x^2 = 60^2 + 60^2 - 2(60)(60)\cos 20°$$

$$\vdots$$

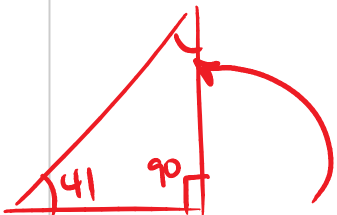
$$x = 21\text{cm}$$

How do we know when to use what?

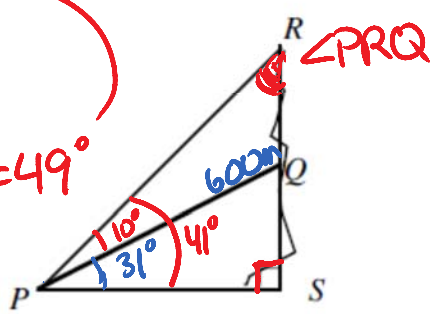
Clues	2 sides, ∠ opposite from one of the sides	2 angles and a side	2 sides and contained angle	3 sides (SSS)
Quick Sketch		180 − ✓✓ 		
Looking for >	angle	side	side	Angle
Use >>	Sine Law	Sine Law (after finding 3rd angle)	Cosine Law	Cosine Law

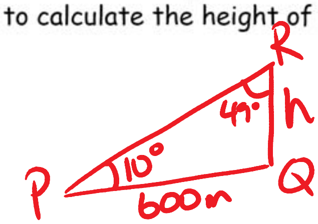
Example 1: P and Q are two bases for a mountain climb. PQ is 600m and QR is a vertical stretch of a rock face. The angle of elevation of Q from P is 31°, and the angle of elevation of R from P is 41°.

- Mark these measurements on the diagram and state the measure of ∠PRQ.
- Use sine law in ΔPQR to calculate the height of the vertical climb, QR, to the nearest metre.



$180-90-41=49°$

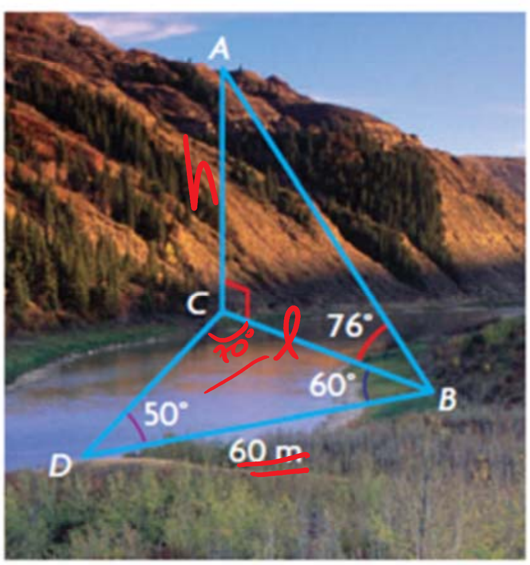




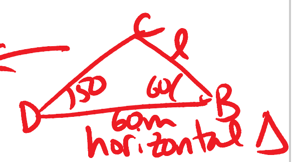
$$\frac{h}{\sin 10°} = \frac{600}{\sin 49°}$$

$$h = \frac{600 \sin 10°}{\sin 49°} = 138\text{ m}$$

Example 2: Brendan & Diana plan to climb the cliff at Dry Island Buffalo Jump, Alberta. They need to know the height of the climb before they start. Brendan stands at point B, as shown in the diagram. He uses a clinometer to determine ∠ ABC, the angle between the base of the cliff, himself, and Diana, who is standing at point D. Diana estimates ∠CDB, the angle between the base of the cliff, herself, and Brendan. Determine the height of the cliff to the nearest metre.

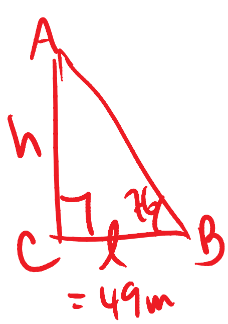


$\angle DCB = 180-50-60 = 70°$



$$l: \quad \frac{l}{\sin 50°} = \frac{60}{\sin 70°}$$

$l = 49\text{m}$ ← base of vertical Δ as well



$$\tan 76 = \frac{h}{49}$$

$$49 \tan 76° = h$$

$$h = 197\text{m}$$

Pg 147 # 3, 4, 6, 7

angle of depression

angle of elevation

Problems Involving Acute Triangles - Part 2 [3.4]

Example 1: Determine the angle of depression, to the nearest degree, for each camera.

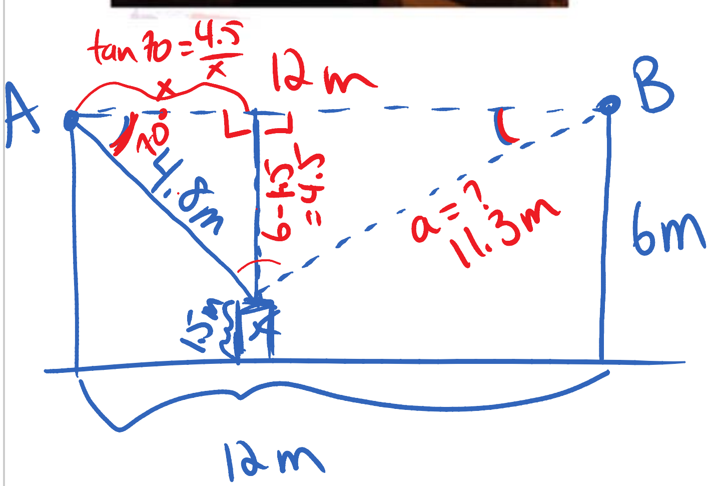
Two security cameras in a museum must be adjusted to monitor a new display of fossils. The cameras are mounted 6 m above the floor, directly across from each other on opposite walls. The walls are 12 m apart. The fossils are displayed in cases made of wood and glass. The top of the display is 1.5 m above the floor. The distance from the camera on the left to the centre of the top of the display is 4.8 m. Both cameras must aim at the centre of the top of the display.

SOHCAHTOA

① $\sin A = \frac{4.5}{4.8}$

$A = \sin^{-1}\left(\frac{4.5}{4.8}\right)$

$= 70^\circ$ = angle of dep of the left camera



② 12, 70, 4.8, a

$a^2 = 12^2 + 4.8^2 - 2(12)(4.8)(\cos 70^\circ)$

$a^2 = 127.639$

$a = 11.3\text{ m}$

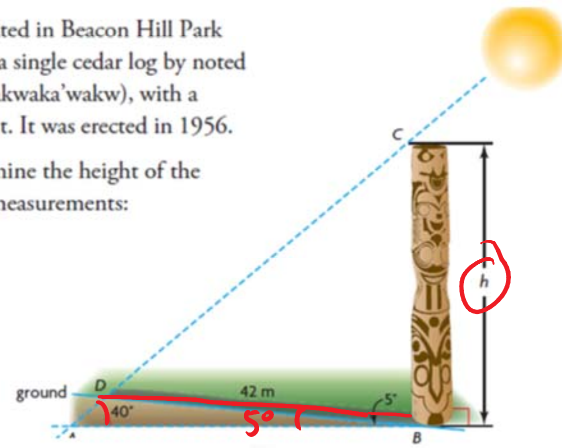
③ $\sin B = \frac{4.5}{11.3}$

$B = 23^\circ$ = angle of dep. of right camera

Example 2:

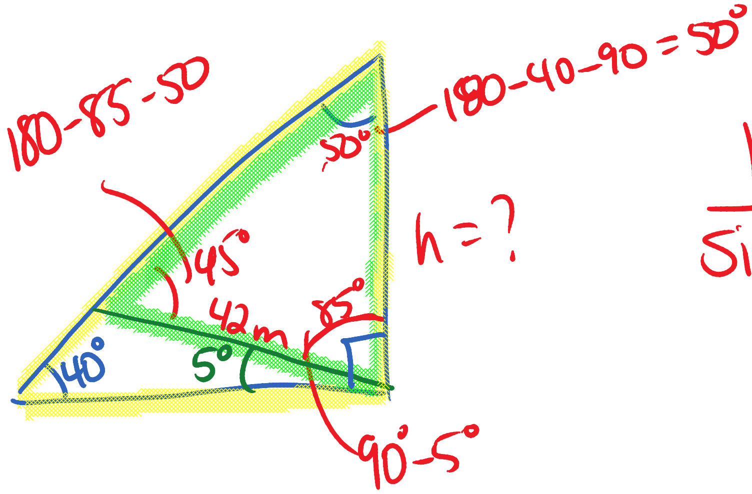
The world's tallest free-standing totem pole is located in Beacon Hill Park in Victoria, British Columbia. It was carved from a single cedar log by noted carver Chief Mungo Martin of the Kwakiutl (Kwakwaka'wakw), with a team that included his son David and Henry Hunt. It was erected in 1956.

While visiting the park, Manuel wanted to determine the height of the totem pole, so he drew a sketch and made some measurements:



- I walked along the shadow of the totem pole and counted 42 paces, estimating each pace was about 1 m.
- I estimated that the **angle of elevation** of the Sun was about 40°.
- I observed that the shadow ran uphill, and I estimated that the angle the hill made with the horizontal was about 5°.

How can Manuel determine the height of the totem pole to the nearest metre?



$$\frac{h}{\sin 45°} = \frac{42m}{\sin 50°}$$

$h = 39m$

pg 147 # 11-14

Pod Racing Diagram

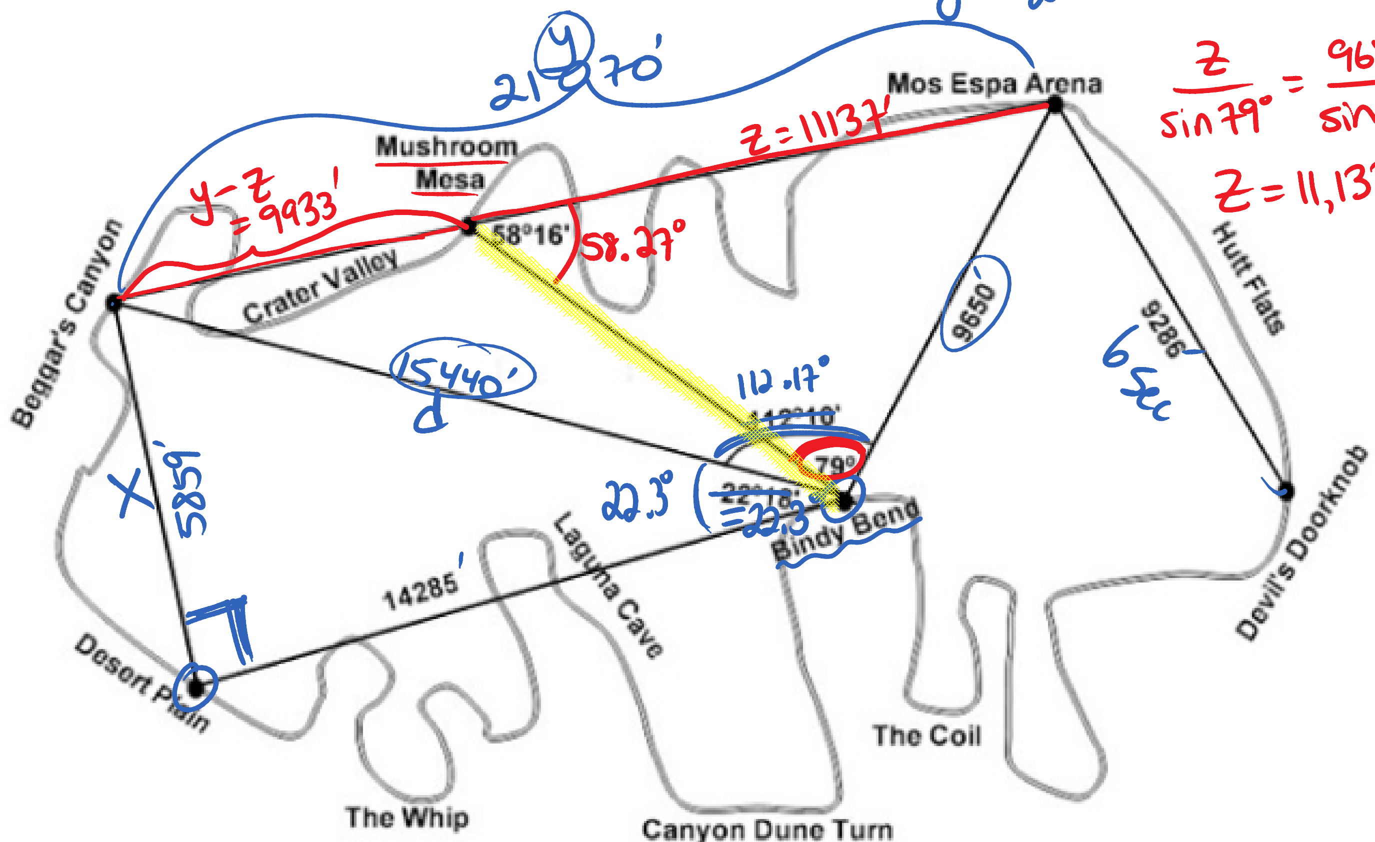
Tuesday, October 01, 2013
12:00 PM

$y^2 = 15440^2 + 9650^2 - 2(15440)(9650)\cos 112.17$

$y = 21070$

$\frac{z}{\sin 79^\circ} = \frac{9650}{\sin 58.27}$

$z = 11{,}137'$



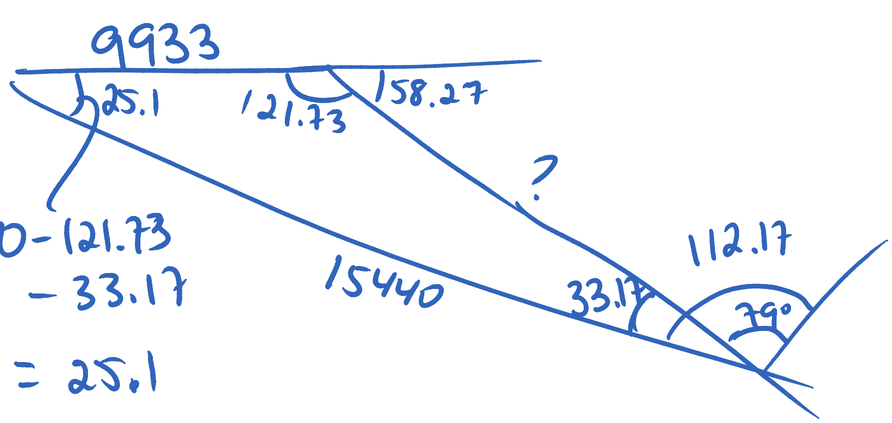
1a) $\tan 22.3^\circ = \frac{x}{14285}$

$x = 5859'$

b) $d^2 = 5859^2 + 14285^2$

$d = 15440'$

3c)



$180 - 121.73$

$- 33.17$

$= 25.1$

$\frac{?}{\sin 25.1} = \frac{9933}{\sin 33.17}$

$? = 7699'$